TWO-DIMENSIONAL INTEGRAL EQUATIONS IN APPLIED MECHANICS OF DEFORMABLE SOLIDS

V. M. Aleksandrov and A. V. Manzhirov

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A study is made of two-dimensional integral equations arising in contact problems for bodies with complex properties, problems involving wear and tear of elastic and viscoelastic bodies. Methods of solution presented are based on the study of nonclassical spectral properties of integral operators. Domains in which these methods can be applied effectively are examined. Theoretical results are illustrated with an applied example.

1. We consider the integral equation

$$c(t)(\mathbf{I}-\mathbf{L}_1)q(x,t)+(\mathbf{I}-\mathbf{L}_2)\mathbf{A}q(x,t)=\delta(t)+\alpha(t)x-g(x),$$
$$(\mathbf{I}-\mathbf{L}_i)f(t)=f(t)-\int_1^t f(\tau)K_i(t,\tau)\,d\tau\quad(i=1,2),\tag{1.1}$$
$$\mathbf{A}f_1(x)=\int_{-1}^{1} f_1(\xi)k(\xi,x)\,d\xi,\quad k(\xi,x)=k(-\xi,-x)$$

subject to the additional conditions

$$P(t)=\int_{-1}^{1} q(x,t)\,dx,\quad M(t)=\int_{-1}^{1} q(x,t)\,x\,dx.\tag{1.2}$$

Here $c(t) > 0$, $\delta(t)$, $\alpha(t)$, $P(t)$, $M(t) \in C[1, T]$; $K_i(t, \tau)$ is a Volterra kernel [1]; $q(x, t)$ is a function continuous with respect to t in $L_2[-1, 1]$; $\mathbf{A}$ is an operator which is completely continuous, self-adjoint, and positive-definite from $L_2[-1, 1]$ into $L_2[-1, 1]$, where $\int_{-1}^{1}\int_{-1}^{1} k^2(\xi,x)\,d\xi dx<\infty$.

We decompose the solution into a sum of parts, even and odd with respect to x, identified, respectively, with subscripts 1 and 2; Eqs. (1.1) and (1.2) then assume the form

$$c(t)(\mathbf{I}-\mathbf{L}_1)q_i(x,t)+(\mathbf{I}-\mathbf{L}_2)\mathbf{A}_iq_i(x,t)=\delta_i(x,t),$$
$$\mathbf{A}_if_1(x)=\int_{-1}^{1} f_1(\xi)k_i(\xi,x)\,d\xi\quad(i=1,2),\tag{1.3}$$

$$\delta_i(x,t)=\begin{cases}\delta(t)-g_1(x), & i=1,\\ \alpha(t)x-g_2(x), & i=2;\end{cases}$$
$$P(t)=\int_{-1}^{1} q_1(x,t)\,dx,\quad M(t)=\int_{-1}^{1} q_2(x,t)\,x\,dx.\tag{1.4}$$

In Eqs. (1.3) and (1.4) let us assume that all the functions are known except for $q_i(x, t)$, $\delta(t)$, $\alpha(t)$. We now find these functions. We study an even version of the problem, putting $i = 1$ in Eqs. (1.3) and (1.4). We note that

$$k_1(\xi,x)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} r_{2m2n}P^*_{2m}(x)P^*_{2n}(\xi),\tag{1.5}$$

where $\{P_m^*\}$ is a complete orthonormalized system of Legendre polynomials in $L_2[-1, 1]$.

We now introduce a complete Hilbert space of even functions, square-integrable on the interval [−1, 1], such that the integral of these functions over this interval is equal to zero. Denote this space by $L_2{}^0[-1, 1]$.

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THEOREM 1. The kernel $k_1(\xi, x)$ is representable in the form [see Eq. (1.5)]

$$k_1(\xi, x) = k_1^0(\xi, x) + 2^{-1/2}k_2^0(\xi) + 2^{-1/2}k_2^0(x) + D/2,$$

$$\int_{-1}^{1} k_1^0(\xi, x)\,d\xi = 0, \quad k_2^0(\xi) = \sum_{n=1}^{\infty} r_{02n}P_{2n}^*(\xi) \in L_2^0[-1, 1],$$

$$k_1^0(\xi, x) = k_1^0(x, \xi) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} r_{2m2n}P_{2m}^*(x)\,P_{2n}^*(\xi), \; D = r_{00},$$

where the operator $\mathbf{B}_1\left(\mathbf{B}_1\varphi(x) = \int_{-1}^{1}\varphi(\xi)\,k_1^0(\xi, x)\,d\xi\right)$ is completely continuous, self-adjoint, and positive-definite from $L_2^0[-1, 1]$ into $L_2^0[-1, 1]$, and the sequence of its characteristic functions ψ_{2i}, corresponding to the characteristic numbers β_{2i} ($i = 1, 2, \ldots$), constitutes a basis in $L_2^0[-1, 1]$.

We can show that the sequence $\{\psi_{2i}\}$ ($i = 0, 1, \ldots; \psi_0 = 2^{-1/2}$) is a complete orthonormalized system of even functions in $L_2[-1, 1]$. We write the solution in the form

$$q_1(x, t) = \sum_{i=0}^{\infty} z_{2i}(t)\,\psi_{2i}(x),$$
$$g_1(x) = \sum_{i=0}^{\infty} g_{2i}\psi_{2i}(x), \quad k_2^0(x) = \sum_{i=1}^{\infty} k_{2i}\psi_{2i}(x). \tag{1.6}$$

Substituting the solution (1.6) into Eqs. (1.3) and (1.4), and taking Theorem 1 into account, we obtain

$$z_{2i}(t) = f_{2i}(t) + \int_{1}^{t} f_{2i}(\tau)\,R_{2i}^0(t, \tau)\,d\tau \quad (i = 1, 2, \ldots),$$
$$f_{2i}(t) = -(c(t) + \beta_{2i})^{-1}[k_{2i}(\mathbf{I} - \mathbf{L}_2)z_0(t) + g_{2i}],$$
$$\delta(t) = 2^{-1/2}\Big[c(t)(\mathbf{I} - \mathbf{L}_1)\,z_0(t) + D(\mathbf{I} - \mathbf{L}_2)\,z_0(t) +$$
$$+ \sum_{i=1}^{\infty} k_{2i}(\mathbf{I} - \mathbf{L}_2)\,z_{2i}(t) + g_0\Big], \quad z_0(t) = 2^{-1/2}P(t),$$

where $R_i^0(t, \tau)$ is the resolvent of the kernel $K_i^0(t, \tau) = [c(t)K_1(t, \tau) + \beta_i K_2(t, \tau)]/[c(t) + \beta_i]$.

2. We now consider an odd version of the problem [with i = 2 in Eqs. (1.3) and (1.4)]. We introduce a complete Hilbert space of odd functions, square-integrable on the interval [−1, 1], such that the integral of the product of a function and x over this interval is equal to zero. Denote this space by $L_2^1[-1, 1]$.

THEOREM 2. The kernel $k_2(\xi, x)$ is representable in the form

$$k_2(\xi, x) = k_1^1(\xi, x) + (3/2)^{1/2}\,\xi k_2^1(x) + (3/2)^{1/2}\,x k_2^1(\xi) + (3/2)\,Cx\xi,$$

$$\int_{-1}^{1}\xi k_1^1(\xi, x)\,d\xi = 0, \quad k_2^1(\xi) = \sum_{n=1}^{\infty} r_{12n+1}P_{2n+1}^*(\xi) \in L_2^1[-1, 1],$$

$$k_1^1(\xi, x) = k_1^1(x, \xi) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} r_{2m+1\,2n+1}P_{2m+1}^*(x)\,P_{2n+1}^*(\xi), \quad C = r_{11},$$

where the operator $\mathbf{B}_2\left(\mathbf{B}_2\varphi(x) = \int_{-1}^{1}\varphi(\xi)\,k_1^1(\xi, x)\,d\xi\right)$ is completely continuous, self-adjoint, and positive-definite from $L_2^1[-1, 1]$ into $L_2^1[-1, 1]$, and the sequence of its characteristic functions ψ_{2i+1}, corresponding to the characteristic numbers β_{2i+1} ($i = 1, 2, \ldots$), forms a basis in $L_2^1[-1, 1]$.

We can show that the sequence $\{\psi_{2i+1}\}$ ($i = 0, 1, \ldots; \psi_1 = (3/2)^{1/2}x$) is a complete orthonormalized system of odd functions in $L_2[-1, 1]$. We write the solution in the form

$$q_2(x, t) = \sum_{i=0}^{\infty} z_{2i+1}(t)\,\psi_{2i+1}(x), \tag{2.1}$$

$$g_2(x) = \sum_{i=0}^{\infty} g_{2i+1}\psi_{2i+1}(x), \quad k_2^1(x) = \sum_{i=1}^{\infty} k_{2i+1}\psi_{2i+1}(x). \tag{2.1}$$

Substituting Eqs. (2.1) into Eqs. (1.3) and (1.4) and taking Theorem 2 into account, we find

$$z_{2i+1}(t) = f_{2i+1}(t) + \int_1^t f_{2i+1}(\tau) R_{2i+1}^0(t, \tau)\, d\tau \quad (i = 1, 2, \ldots),$$

$$f_{2i+1}(t) = -(c(t) + \beta_{2i+1})^{-1}[k_{2i+1}(\mathbf{I} - \mathbf{L}_2) z_1(t) + g_{2i+1}],$$

$$\alpha(t) = (3/2)^{1/2}\Big[c(t)(\mathbf{I} - \mathbf{L}_1) z_1(t) + C(\mathbf{I} - \mathbf{L}_2) z_1(t) +$$

$$+ \sum_{i=1}^{\infty} k_{2i+1}(\mathbf{I} - \mathbf{L}_2) z_{2i+1}(t) + g_1\Big], \quad z_1(t) = (3/2)^{1/2} M(t).$$

3. We consider now the case in which q(x, t), P(t), and M(t) in Eqs. (1.3) and (1.4) are unknown. Since in obtaining the solution we employ the classical procedure of the spectral theory of operators [1, 3], we supply only the basic formulas. We take the solution in the form

$$q_1(x, t) = \sum_{i=0}^{\infty} \omega_{2i}(t)\varphi_{2i}(x), \quad g_1(x) = \sum_{i=0}^{\infty} g_{2i}^*\varphi_{2i}(x),$$

$$q_2(x, t) = \sum_{i=0}^{\infty} \omega_{2i+1}(t)\varphi_{2i+1}(x), \quad g_2(x) = \sum_{i=0}^{\infty} g_{2i+1}^*\varphi_{2i+1}(x),$$

$$1 = \sum_{i=0}^{\infty} \delta_{2i}\varphi_{2i}(x), \quad x = \sum_{i=0}^{\infty} X_{2i+1}\varphi_{2i+1}(x),$$

$$\mathbf{A}\varphi_i = \alpha_i\varphi_i \quad (i = 0, 1, 2, \ldots).$$

Here α_i are characteristic numbers of the operator **A** and φ_i are its orthonormalized characteristic functions. We substitute Eq. (3.1) into Eqs. (1.3) and (1.4); then

$$\omega_i(t) = \Delta_i(t) + \int_1^t \Delta_i(\tau) R_i^*(t, \tau)\, d\tau,$$

$$\Delta_i(t) = \begin{cases} [\delta_i\delta(t) - g_i]\, d_i^{-1}(t), & i = 0, 2, \ldots, \\ [\alpha(t) X_i - g_i]\, d_i^{-1}(t), & i = 1, 3, \ldots, \end{cases}$$

$$P(t) = \sum_{i=0}^{\infty} \delta_{2i}\omega_{2i}(t), \quad M(t) = \sum_{i=0}^{\infty} X_{2i+1}\omega_{2i+1}(t),$$

where $R_i^*(t, \tau)$ is the resolvent of the kernel $K_i^*(t, \tau) = [c(t)K_1(t, \tau) + \alpha_i K_2(t, \tau)]d_i^{-1}\cdot(t)$, $d_i(t) = c(t) + \alpha_i$.

It can be shown that in the class of functions chosen the solution of Eq. (1.3), subject to the conditions (1.4), exists, is unique, and may be constructed by the methods set forth with an arbitrarily high degree of accuracy.

We mention now several areas of application of the proposed methods: 1) contact problems of the theory of elasticity [4] and viscoelasticity for bodies with roughnesses and coatings subject to wear and tear; 2) contact problems for nonhomogeneous viscoelastic aging laminated foundations [5, 6]; 3) contact problems for wear and tear of viscoelastic foundations with complex rheologies. Similar equations occur in [7, 8].

As an example we consider a two-dimensional contact problem concerning frictionless impression $Q(t^0)$ of a rigid stamp on the surface of a two-layered foundation: a nonhomogeneously aging thin layer [8, 9], and a homogeneously aging layer of arbitrary thickness H. The thin layer of thickness h lies frictionless on the homogeneously aging layer, coupled with a nondeforming foundation. The stamp has a flat bottom, a contact line of length 2a, and off-center application of force equal to zero. Based on [9-11], we arrive at an even version of Eq. (1.1) subject to a given supplementary condition (see also [5, 6]), where

$$t = t^0\tau_1^{-1}, \quad \tau = \tau^0\tau_1^{-1}, \quad x = x^0 a^{-1}, \quad \delta(t) = \delta^0(t^0)\, a^{-1},$$

$$\alpha(t) = 0,\ g(x) = 0,\ M(t) = 0,\ Ha^{-1} = \lambda,$$

$$k(\xi, x) = \pi^{-1}k^0[(\xi^0 - x^0)/H], \quad \xi = \xi^0 a^{-1},$$
$$\varkappa(y) = \varkappa^0(y)\tau_1^{-1}, \; q(x, t) = q^0(x^0, t^0)\theta_2^{-1}(t^0 - \tau_2),$$
$$K_2(t, \tau) = K_2^0(t^0 - \tau_2, \tau^0 - \tau_2)\tau_1, \quad K_1(t, \tau) = \theta_2(\tau^0 - \tau_2)\theta_1(t^0)\theta_2^{-1} \quad (t^0 - \tau_2)\theta_1^{-1}(\tau^0)\tau_1 K_1^*(t^0, \tau^0),$$
$$P(t) = Q(t^0)a^{-1}\theta_2^{-1}(t^0 - \tau_2), \quad c(t) = 0{,}5ha^{-1}\theta_2(t^0 - \tau_2)\theta_1^{-1}(t^0),$$
$$K_1^*(t^0, \tau^0) = \frac{1}{h}\int_0^h K_1^0(t^0 + \varkappa^0(y), \; \tau^0 + \varkappa^0(y))\,dy,$$
$$K_i^0(t^0, \tau^0) = E_i(\tau^0)\frac{\partial}{\partial\tau^0}\left[\frac{1}{E_i(\tau^0)} + C_i(t^0, \tau^0)\right] \quad (i = 1, 2),$$
$$[E_1^*(t^0)]^{-1} = \frac{1}{h}\int_0^h E_1^{-1}(t^0 + \varkappa^0(y))\,dy,$$
$$\theta_1(t^0) = \frac{E_1^*(t^0)}{2(1 - \nu_1^2)}, \quad \theta_2(t^0) = \frac{E_2(t^0)}{2(1 - \nu_2^2)}.$$

Here t^0 is the present instant of time; τ^0 is the variable of integration on a real time scale; x^0 is the horizontal coordinate; $\delta^0(t^0)$ defines the settling under the stamp; $K^0 \cdot [(\xi^0 - x^0)/H]$ is the kernel of the two-dimensional contact problem; ξ^0 is the variable of integration over a real scale of lengths; $\kappa^0(y)$ is a function of homogeneous aging through the depth of the thin layer; $q^0(x^0, t^0)$ is a function of contact pressures under the stamp; τ_1 is the age of the elements of the lower boundary of the thin layer at the instant of application of the load; $K_i^0(t^0, \tau^0)$, $C_i^0(t^0, \tau^0)$, $E_i(\tau^0)$, and ν_i are, respectively, kernels, measures of creep, moduli of elastoinstantaneous deformation, and Poisson coefficients of the top ($i = 1$) and bottom ($i = 2$) layers; τ_2 is the preparation time for the bottom layer.

We write the measures of creep in the form [11]

$$C_i(t^0, \tau^0) = \varphi_i(\tau^0)f_i(t^0 - \tau^0), \quad \varphi_i(\tau^0) = C_i^0 + A_i^0 e^{-\beta_i^0\tau^0},$$
$$f_i(t^0 - \tau^0) = \left(1 - e^{-\gamma_i^0(t^0 - \tau^0)}\right), \quad \beta_i = \beta_i^0\tau_1, \; \gamma_i = \gamma_i^0\tau_1.$$

As the material of the layers we take concrete; then, assuming the elastic characteristics to be fixed, we take the following values for the parameters [11, 12] (since the materials of the layers are identical, we omit subscripts on the parameters): $c(t) = 0.2$, $\lambda = 2$, $C^0E = 0.5522$, $A^0E = 4$, $\tau_2 = 0$, $\nu = 0.3$, $\beta^0 = 0.031$ day^{-1}, $\gamma^0 = 0.06$ day^{-1}. We consider the cases of natural and artificial nonhomogeneous aging of a bundle of layers following [6, 12]. As the total characteristic of nonhomogeneous aging, we introduce the nonhomogeneous aging parameter $\mu = \frac{1}{h}\int_0^h e^{-\beta\varkappa(y)}dy$.

1. Natural nonhomogeneous aging

$$1 \leqslant \mu < e^{\beta}, \; \tau_1 = 75 \text{ days}, \; P(t) = 0.5(1 - \cos 4\pi t).$$

2. Artificial nonhomogeneous aging

$$0 < \mu \leqslant 1, \; \tau_1 = 10 \text{ days}, \; P(t) = 0.5[1 - \cos 0.4\pi(t - 1)].$$

The age τ_1 differs for the two aging cases; therefore, on the graphs we give two dimensionless time scales t, where the functions P(t) are chosen so that in terms of these scales the loads are specified by one and the same curve. We examine separately the process in which $\mu = 1$ in cases 1 and 2, which corresponds to homogeneous aging of the foundations. We note, by virtue of [8, 9], that

$$k^0(z) = \int_0^\infty \frac{L(u)}{u}\cos uz\,du,$$
$$L(u) = \frac{2\varkappa \operatorname{sh} 2u - 4u}{2\varkappa \operatorname{ch} 2u + 4u^2 + 1 + \varkappa^2}, \quad \varkappa = 3 - 4\nu.$$

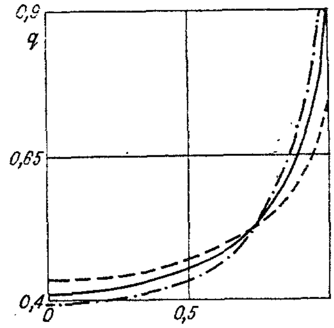

Fig. 1

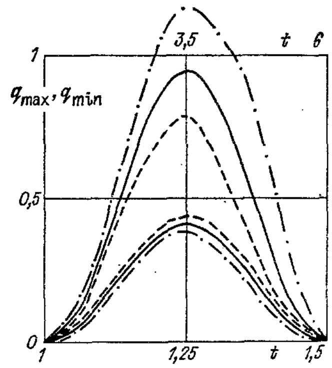

Fig. 2

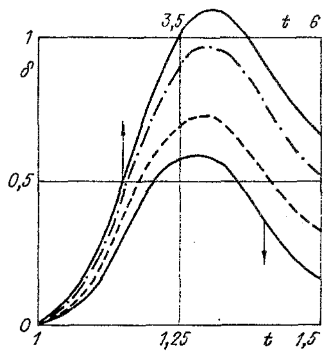

Fig. 3

We denote the characteristics for the homogeneous ($\mu = 1$), the natural nonhomogeneous ($\mu = 10$), and the artificial nonhomogeneous ($\mu = 0.05$) processes of aging of the foundation by continuous, dash, and dot–dash curves, respectively. The dash curves always refer to the lower time scale, while the dot–dash curves refer to the upper time scale.

Figure 1 shows the dependence of $q(x, t)$ on x: homogeneous aging for arbitrary t, for which $P(t) = 1$; natural nonhomogeneous aging for $t = 1.25$; artificial nonhomogeneous aging for $t = 3.5$. It is evident that for natural nonhomogeneous aging the distribution of contact pressures becomes more uniform in comparison with the case of homogeneous aging of the bundle of layers; for artificial nonhomogeneous aging the picture is just the reverse.

Figure 2 shows the variation of the maximum and the minimum contact pressures with respect to the time, depending on the type of aging. This expresses the fact that in the case of homogeneous aging the contact pressures depend only on the magnitude of the load acting on the stamp with a flat bottom, i.e., creep has no effect on their distribution and, particularly, on the elastic solution. Naturally, for loads symmetric with respect to some instant of time, the continuous curves are symmetric. For nonhomogeneous aging, for the very same loads, symmetry for the analogous curves is violated.

The trend in the variation of q_{max} and q_{min} is easily determined from the figure. However, in the case of natural nonhomogeneous aging, at each time instant t, with $P(t) \neq 0$, the maximum contact pressures are always less, and the minimum contact pressures are always greater, than the analogous pressures in the case of homogeneous aging. The reverse conclusions apply to the case of artificial nonhomogeneous aging.

Figure 3 shows the dependence of settling under the stamp, $\delta(t)$, on the time. The two continuous curves show that $\delta(t)$, for $\mu = 1$, depends on the rheological properties of the foundation, in particular, on the choice of τ_1. When $t > 1$, the magnitude of settling, in the case of natural nonhomogeneous aging is always greater, and in the case of artificial nonhomogeneous aging is always less, than its magnitude for the homogeneous version.

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GENERALIZED THEORY FOR NONISOTHERMAL STRAIN

É. I. Blinov and K. N. Rusinko

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The problem is solved for analytical description of relationships between strains, stresses, and temperature "at a point" of a solid during its thermomechanical loading. Stresses are divided into equilibrium and nonequilibrium. Equilibrium stresses do not depend on time effects for deformation, and through them by methods of classical plasticity theory irreversible strain is determined. Recovery of mechanical properties at elevated temperatures is considered. Apart from nonisothermal plastic strain, nonsteady and high-temperature steady-state creep and thermal aftereffect are described.

1. Basic Assumptions. Consideration is given to the condition of a constant density substance in the quite small neighborhood of a point of deforming solid as an element characterizing the condition at this point. Due to the specific nature of strains for a solid, this phenomenological element of material forms a closed thermodynamic system in which classical thermodynamic laws operate [1]. Experiments show [2, 3] that if at a certain instant of thermomechanical loading for an actual solid strain and temperature are fixed, then after this there is a reduction in stress so that complete (thermodynamic) equilibrium in the system sets in only after this relaxation.

Stresses which remain after transfer of the system from an actual to a thermodynamic equilibrium condition we call equilibrium, and the relaxed part of stresses whose tensor components are obtained by subtracting the equilibrium stress tensor component $\langle\sigma_{ij}\rangle$ from the corresponding stress tensor component σ_{ij} occurring at the instant of fixing strains and temperature, we call nonequilibrium stresses and we designate them ψ_{ij}. Thus, in each instant of deformation there is an identity

$$\psi_{ij} \equiv \sigma_{ij} - \langle\sigma_{ij}\rangle,\ i,\ j = 1,\ 2,\ 3. \tag{1.1}$$

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